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USING A MANAGERIAL ANALOGY FOR MAKING MATHEMATICS MORE ATTRACTIVE

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Abstract: In many countries, Mathematics is among the least popular school subjects. Educators are spending a lot of time in searching methods that could change this unfortunate situation. The author, a lecturer at a School of Management, discusses the issue as a managerial problem.

First, the popularity of mathematics is analyzed as a marketing problem. Marketing theories recommend increasing the market choice as a way of attracting more customers. Our first goal is therefore to expand the variety of problems. To do so, we consider Knowledge Management concepts to demonstrate an imbalance between tacit and explicit knowledge in traditional courses. Currently used educational methodologies favour the latter one. Examples of the approaches that might increase learners' tacit knowledge are our second goal.

Keywords: popularity of Mathematics; marketing problem.

1. Introduction

In many countries, Mathematics belongs among the least popular school subjects. Tapia and Marsh [Tapia & Marsh, 2004] give a large overview of research targeting students' and pupils' attitude to the subject. Their paper conclusions confirm the ambivalent attitude: on one side, the students are aware of value of mathematics, on the other side, they are not self-confident in doing it and motivated to progress. The reasons for the low popularity of mathematics sometimes reach absurd dimensions [Swan, 2004] but in general they have their roots an absence of understanding of the role of Mathematics in their future life i.e. in *reasons to learn*.

The author, a lecturer of School of Management, discusses the issue as a managerial problem. Using analogy, one can consider the "absence of interest in learning" as a special case of "prospective addressees' loss of interest in a service offered to them". Under this perspective, Mathematics is a *product* offered by teachers to its "clients" – pupils and students – and through them to the entire society. The community of Mathematics educators happens to be in the situation of a company losing its position on its market. In [Hvorecký, 2007], an example of disappearance of an "educational product" from "educational market" is shown. Latin, the lingua franca of all educated people in medieval Europe, is no more offered as a required high-school subject. To prevent a similar threat to mathematics, measures should be taken.

Managerial theories recommend two principal improvement methods:

- Internal methods are based on innovation of production processes. From one side, the production can be performed with fewer costs in the hope that the client is ready to buy the cheaper product. In our case, it means making changes in educational methodology leading to more effective and efficient classroom communication in order to give to the students as much mathematically-oriented knowledge as possible. This approach is typical for research in Teaching Mathematics.

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From the managerial point of view, the investment must bring back noticeable benefits otherwise it is wasted. The author's personal feeling is that the outcome does not correspond to the efforts invested into the research as the youngsters' interest in Mathematics does not grow in any measurable proportions.

- External methods are based on responding to client's needs. Such an approach presumes accepting teaching mathematics as a service to public and introducing added values into the product in order making it more attractive to it. It leads to redesigning the product, to making its substantial innovation relevant to changed social/economic/cultural conditions.

  Similar changes are our priority. New technology (including hand-held calculators) is capable of solving advanced problems (e.g. symbolic algebraic manipulations and animated geometrical constructions). These key components of the traditional content of Mathematics become "old-fashioned". As they require a lot of concentration and punctuality in their performance, they substantially contribute to the low popularity of mathematics. As this part of training can now be reduced, we may discuss relevant enrichment of Mathematics education methodologies that will reflect the change.

In this paper, we concentrate on specifying an innovative substitute. We ask what fields of mathematical knowledge could make it more "customer-oriented' and, at the same time, develop their mathematical thinking and reasoning. We apply two basic concepts of Knowledge Management – tacit and explicit knowledge. As explicit knowledge can be executed by technology, people should concentrate on tacit knowledge because it only resides in human's brains. We study the ways what to emphasize and how.

2. Tacit and explicit knowledge

Tacit and explicit knowledge are key concepts of knowledge management – a new brand of management that tries to make production more efficient and effective by improving internal (in our interpretation, educational) processes by capturing and storing all relevant knowledge and disseminate it within the organization to enhance its performance and competitiveness [Business Dictionary, 2009]. Knowledge is present in two forms [OR Society, 2003]:

- Explicit knowledge is codified, precisely and formally articulated. As such, it is rather easy to codify, document, transfer, share, and communicate. It is stored using various media like books, journals, video-records, and similar.
- Tacit knowledge is subconsciously understood or applied, difficult to articulate, developed from direct action and experience, shared through conversation, story-telling etc. It is exclusively stored in human brains. It is rather difficult to share as the observers basically see its application, not its elements.

The difference between them has been presented for example in Prof. Back's lecture at the CADGME 2009 conference in Hagenberg, Austria [Back 2009]. He showed that formal manipulations require both tacit and explicit knowledge. Traditionally, the process of simplification of an expression is viewed in the following way:

$$\tan\frac{17\pi}{3} = \tan\left(\frac{6.2\pi}{3} + \frac{5\pi}{2}\right) = \tan\left(2.2\pi + \frac{5\pi}{2}\right) = \tan\frac{5\pi}{2}$$

Here we replicate the same simplification using Back's notation. The curly brackets between the traditional steps contain elements of solver's tacit knowledge – considerations leading to the next formula.

$$\tan\frac{17\pi}{3} =$$

{Factor out 2π}

$$\tan\left(\frac{6.2\pi}{3}+\frac{5\pi}{2}\right)=$$

{Write the angle in the form 2π + α}

$$\tan\left(2.2\pi+\frac{5\pi}{2}\right)=$$

{Neglect factors of 2 π}

$$\tan\frac{5\pi}{2}$$

In reality, the situation is even more complex. For example, the first tacit element (*Factor out 2π*) and the third tacit element (*Neglect factors of 2π*) together form another, more complex element of tacit knowledge as the initial factorization is done purposefully. This meta-element suggests: *Before you start, look for your opportunities to simplify*.

Our tacit knowledge also tells us when to end: *Is the last expression the final one or is further simplification expected?* Notice that there is not a unique answer to it. The right one depends on the context. Tacit knowledge may help us to recognize it. In accordance to Back, educators expect their students to show the formal manipulations but their reasoning and argumentation often remains hidden. Also, teachers' perception is subjective. At the same time, they have power to decide on the presumed end of simplification. As a result, students may pretend conforming teachers' desires to their own judgment.

3. Knowledge transfer

The necessity of combining both tacit and explicit knowledge has been stressed by other authors, too. For example, [Wilson & Cole, 1991] state that the content should consist of tacit, heuristic knowledge as well as textbook knowledge. Nonaka and Takeuchi, founders of Knowledge Management, described the knowledge transfer using the SECI model – see Figure 1. SECI refers to four principal activities necessary for gaining and sharing knowledge: Socialization, Externalization, Combination, Internalization (12Manage, 2009).

	Tacit knowledge	Explicit knowledge
Tacit knowledge	**Socialization**	**Externalization**
Explicit Knowledge	**Internalization**	**Combination**

Figure 1. *The SECI Model*

Socialization means (verbal and non-verbal) human-to-human communication in which partners learn from each other by idea exchange, observation, and cooperation. They may even not be aware that they are getting new knowledge. The gained knowledge remains in persons' heads. For getting person's feedback, discussions are an appropriate method of control as the person knowing something is capable of talking about it, even if using fuzzy, descriptive terms.

Externalization is the process in which one is trying to make his/her ideas and concept "visible" using an appropriate notation or a form of expression. The target notation has to be "standardized". Otherwise, the message put into it could be misunderstood or entirely lost by others. The ability to externalize person's tacit knowledge can be verified by consistency between ideas and their formal

expression. Above Back's lecture shows an attempt to visualize the tacit knowledge necessary for the expression simplification.

Combination is based on manipulations using a standard notation. The manipulations may lead to new, previously unknown knowledge. The properly applied manipulations should lead to correct fact. In fact, this is the reason why such formal notations have been invented. The main verification method is the practical applicability of the result.

Internalization helps us to understand the results of the Combination outcomes. Manipulations are often so complex that we have to study and interpret their results in order to understand "what we have done". Such control is of a special importance today when technology can perform them. By their interpretation, we acquire tacit knowledge that is then stored in our memory – and the cycle can repeat.

Learning processes take place at two levels: (1) inside of each of the four quadrants (2) in the clockwise order indicated by their initial letters. The former way of learning can be simulated in mathematical education e.g. by solving word problems. In the unsurpassed case, the problem is raised by students. Their motivation to solve it will become greater; they will understand its roots and sense a gap in their knowledge. The problems born in this way are the "true S-type problems" because they require their "preprocessing" in the learners' brains, for example an exact specification what to solve and what is expected from the result.

The word problems in textbooks begin at the letter E because they were generated by their writers. They might have good reasons for selecting them but the reasons are hidden from students. Still, they require students to "translate" verbally expressed problem into a formal one. It requires them to understand the problem as a piece of tacit knowledge and change it into an explicit one. Such transformation produces an expression, equation, geometric construction or other formalized task. This new notation considerably differs from the original one. To measure the quality of their reasoning, the learners should ask themselves: *Does the outcome properly express my intention?*

Using the standard mathematical operations to manipulate with expressions, equations, geometric constructions and other mathematical objects are activities of the C-type. In school education, this stage is the most frequent. Unfortunately, it is rarely connected to the previous two ones. Due to that, the learners may miss links to their experience and/or previously gained tacit knowledge and feel lost. Many perform them in a perfect way without having an idea of their relation to real-world problems.

I-type problems started appear more frequently with introduction of information technology. The students can "play" with concepts. They can see the influence of change of values of parameters to curves, animate geometric constructions and similar. In this way, they can internalize these explicit bodies. Again, the students' involvement is the key element as they should control their experiments to the features that raise their curiosity.

4. Implementing the SECI model

From the above arguments, one can recognize the importance of the implementation of the SECI-model-style education of mathematics. Nevertheless, its quick implementation is almost impossible to expect in near future. Reasons are obvious:

1. The full transformation of teaching mathematics into this form would require total redesigning of its basics. This would be risky without making serious and thorough long-term experiments.
2. Problems consisting of all four stages would likely be very complex and time-consuming. There is a danger that children would get lost in them. Without "quick successes", they might become less and less interested.
3. Teachers, school administrators, parent and community might be confused by the method. Their initially mild objections would grow into their total resistance. It happened to the New Maths (Kline, 1973).

These objections do not prohibit the introduction of elements of SECI model into mathematical education. In particular, larger accent on tacit knowledge seems to be a must. This approach is not really new. Without knowledge management terminology, it was mentioned meny times. Wilson and Cole's paper for example speaks about providing hints and help (an S activity), thinking about future actions (an E activity), looking back, and analyzing the strategy (i-type activities). The problem is the balance between these processes. Even if we are capable of passing through the entire SECI loop in many cases, traditional educational activities concentrate on C-type activities. As they are dominant in current courses, they often weaken the effects of S-, E-, and I-type activities. For that reason, we give a few examples of the "missing" ones.

4.1. S-type activities

The S-type activities are done during interpersonal communication. They consist of couching, observation of a more experienced person's performance, participation in problem-solving processes, discussions, and similar.

Figure 2. *Canisters with three balls*

Discussions are a typical method of knowledge transfer and form an integral part of all previous activities. The most valued are continuing discussions in which the initial problem evolves. The problem to discuss must be rather simple and interesting. The problems which are posed without using a formal mathematical notation are appropriate candidates. When properly formulated they can lead to advanced mathematical results. The next problem targets the proportion between a sphere and cylinder of the same diameter: *There are 3 containers with tennis balls and an empty one (see Figure 2). Pour water into the canisters with balls until they are full. Hold the top ball to guarantee that water only fills the empty room. Then infuse the water from 3 canisters into the empty one. It will become full. What can you say about the proportion between volume of the canister and balls in it?*

The problem can be solved just using common sense: *Let us presume that the volume of the empty canister is 1 litre. As we need three canisters with balls to fill it to the top, the contribution of each of them is 1/3 litre. The remaining 2/3 of its volume are occupied by the balls.* It is probable that students will raise several hypotheses and develop their correct solution in a stepwise manner. Such a stepwise progress may show them that achieving a mathematical result is not a straightforward way. Some of them may use calculations but they should be assumed as an alternative method.

Coaching is a method of education based on direct instruction and control of students. In classrooms, it appears rather frequently. As the number of the students grows, it becomes less and less efficient because the teacher has not chance to control everyone permanently and individually. A way of eliminating this drawback is solving group problems. Cooperating pairs or groups of more- and less-skilled learners may intensify it by using peer-to-peer coaching.

Observation of a more experienced person's performance also takes place in our classrooms. Again, the teachers are the persons who are dominantly observed. As learners are often affraid to interrupt them and ask for their detailed explanation, the observation can miss its goal. Peer-to-peer teamwork can bring enrichments to these processes as well.

Participating in problem-solving processes may happen in any environment. It presumes that the problem is interesting enough to capture each student's curiosity and easy enough to be solved by him/herself. The probability that a learner will participate in less formal problems (e.g. without formulas) is higher. The participation can evoke the learner's interest in mastering his/her skills – examples are problems like Rubik's cube or Sudoku.

Competition and mastership are often based on the prolongation and generalization of the initial problem: *Tennis balls are sold also in packages of four balls. Let us now consider them. Will the fourth canister suffice for the water poured from three full ones or will one need an additional small container?* One can expand discussions using (non-existing) containers holding 5, 6, 7 balls as well as smaller ones with 2 or 1. The last one allows making the final conclusion: the expression of the the relationship between a sphere and the cylinder with the diameter and height equal to the diameter of the sphere.

As there are always two alternatives, each of them will likely find its supporters. It may lead to heated *discussions* – another key social activity. In the above example, the two groups will likely appear – the proponets and the opponents of the idea "the fourth identical container suffices". For the success of the process it is very important to goive them time for creating their own statements and then to drive a fair discussion in which faulty side will recognize its errors and become voluntarily convinced in the correctness of its partners' statement.

4.2. E-type activities

Any activity that presents our state of mind is a sort of externalization. The outcome can be a sentence, an essay, a drawing, guidelines, etc. In mathematics, we are predominantly interested in outputs using a formal notation. On the other hand, too much formalization repels our learners. For that reason, even if the discussion on the proportion of balls and canisters should be initially done in a natural language, it should aim to more formal expressions. "More formal" does not mean "nothing but formal". For transferring knowledge one can use a semi-formal notation:

$$
\begin{aligned}
&3Cans - 9Balls = 1Can \\
&2Cans = 9Balls \\
&1Can = 4{,}5Balls \\
&EmptySpace = 1{,}5Balls
\end{aligned}
$$

The author derives his suggestion from programming. It has been shown that legibility of the code using mnemonic variable names is much higher (Dahl el al, 1972). Learners will likely understand it better, even if it is not "mathematically perfect" in the traditional meaning.

Another approach allowing students to create relations between a formal notation and real-life situations is their training in "inverse modelling". Models create relations between a person's mental picture and its formal description. For training reasons, we recommend occasional opposite process. For example, many learners have problems with inequalities. During inverse modelling, we pose a formal expression and ask students what it can describe. *What does the inequality*

$$90 + x \geq 120$$

express? There are two of many options:

- Water level in a small creek is quickly rising. It has reached 90 cm. How much it has to rise to overflow through embankment 119 cm high?
- Driving to our cottage usually lasts two hours. We have already been driving one and a half of hours. How many minutes will our journey take?

The discussion should include the necessity to use the “greater or equal to” symbol. The flooding appears at any water level from120 cm up; the journey can get prolonged due to unexpected obstacles like detour or traffic jam. One can quickly recognize that using the equality symbol is too optimistic. The discussions should be guided to this conclusion.

4.3. I-type activities

Humans are unable to operate anything without having tacit knowledge about it. During the I-type activities, our explicit knowledge internalizes and changes into the tacit one. Its aim is to create a mental model inside the learner’s brain. In the future, these models will guide the learners in application of the relevant pieces of their explicit knowledge.

The problem is that the internalization often happens unknowingly – we may not to intend to learn and not be aware that we are learning. (When a non-swimmer falls into water he/she tries to survive not to learn swimming. The new piece of knowledge occurs as a “side-effect” of his/her survival.) To enhance these processes and to support forming correct mental models are the main aims of every education – schooling, apprenticeship, couching, etc. Internalization makes the learners understanding what they are doing and how. One of promising ways is offered by the above Back’s method.

The contribution of modelling [Kadijevich et al, 2005] can be substantial under the presumption that the learners have enough time to play with the models. In particular, computerized models can do it efficiently due to their high speed of evaluation and the opportunity to visualize the outcome of students’ activity. To support internalization, the execution and outcomes of the experiments should be subject of discussion among students. The following example shows two alternative ways of visualizing the concept of limit

$$\lim_{x\to 4}\frac{(x-4)(x+1)}{\sqrt{x}-2}$$

In the first one, students just calculate values of the function (e.g. as a table in a spreadsheet):

x	**f(x)**
3.95	19.73793042
3.96	19.79027538
3.97	19.84265485
3.98	19.89506880
3.99	19.94751719
4.00	
4.01	20.05251718
4.02	20.10506870
4.03	20.15765453
4.04	20.21027463
4.05	20.26292896

The key is to open the discussion on the absent value. The discussion should clarify (a) why the value of *f*(4) cannot be calculated; (b) what value is the “best candidate” for it [Hvorecky, 2006].

Then, the data can be an input to the graph (Figure 3). The graph is interrupted. Again, the graph helps to identify 20 as the value that could be added to the table to make the graph smooth and uninterrupted.

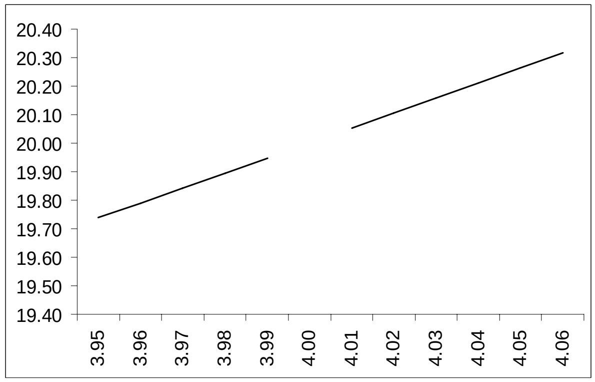

Figure 3. *An interrupted graph*

“Surprising problems” are also a method of internalization as the students have to think deeper on them. An example is: *The formula*

$$\frac{a}{b} * \frac{c}{d} = \frac{a * c}{b * d}$$

is valid. Is the formula

$$\frac{a}{b} : \frac{c}{d} = \frac{a : c}{b : d}$$

also valid?

As students tend to take mathematical knowledge as correct without exemptions, it is wise to undermine their misconception. Take the next popular problem as an example: *The train goes from the town A to town B on the speed v km per hour. The distance between A and B is d kilometres. How many hours will the train travel?* When the problem is solved, ask the students whether the result is valid in all cases. After their approval, pose the question: *Also if A is New York and B London?* This will help them to comprehend that the task is a model with its limits. Their discussion can be heated up by calling e.g. a competition on the most absurd counterexample.

5. Conclusions

Full comprehension of mathematics evidently requires pieces of both explicit and tacit knowledge. At the same time, their weight is different in various categories of problems.

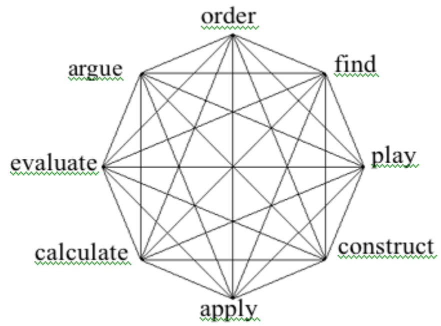

Figure 4. *Zimmerman's octagon*

Figure 5 shows Zimmerman's octagon [Zimmerman, 2003] with 8 activities that have been crucial for the development of mathematics during the history of humankind. Four of them (Find, Construct, Evaluate, and Calculate) stress its "rigid" character – exact, formal, and explicit. The other four (Order, Play, Apply, and Argue) show its "soft" character of an "intellectual game". Despite that fact that tacit and explicit knowledge are inseparable in all of them, the author feels that explicit knowledge dominates in the latter ones while tacit knowledge is leading us in the former one. The feeling is based on the fact that whilst for example Computer Algebra systems successfully solve the prevailing majority of elementary and high school problems, they can hardly express desired outcomes of the other ones:

- *Ordering* should lead to more legible data and respect personal preferences and needs;
- *Play* should be a pleasure;
- *Application* is only possible when the core of the problem is disclosed;
- *Argumentation* must convince the opponents.

In all of these cases, the "truth" is not in correct execution of the task but in the internal acceptance of the process and its outcome. Such acceptance has a little to do with formal verification.

As we mentioned before, a lot of efforts have been done to internal improvements of teaching methodology. Now, the approaches addressing the external audience should also be stressed. "Marketing" of mathematics should concentrate on showing its attractive side. There are two ways of doing so:

- Include additional elements of soft character. Popular puzzles such as *Sudoku* not only look like mathematics, they *are* such because they support the development of mental skills that are truly mathematical.
- Present both "rigid" and "soft" elements and relations between them. Soft components should be accepted as the solid part of mathematical classes.

Stressing the soft character of mathematics does not mean making it less exact. Mathematics is exact and must remain exact. But, at the same time, it should be more attractive for students. There is another managerial analogy behind – marketing must concentrate on features that are valued by the potential clients. Unfortunately, marketing of mathematics is an unsurveyed land.

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